The scale **Journal Website:** https://thescalejournal.com.ng/
An Interdisciplinary Journal of
The ACADEMIC STAFF UNION OF POLYTECHNICS - ZONE D
ISSN: 2992-3727 Published 30th November, 2025

APPLICATION OF TOTAL POTENTIAL ENERGY FUNCTION IN BUCKLING ANALYSIS OF CCCC AND CSCS THIN RECTANGULAR LAMINATED COMPOSITE PLATES

Okoroafor, S. Ugwa [1*].; Anyadiegwu, P. Chibueze [2]
[1,2]Department of Civil Engineering, Federal Polytechnic Nekede, PMB 1036, Owerri, Nigeria.
*Uokoroafor@FPNO.Edu.ng (08037303718)

Abstract
This work applies total potential energy function in buckling analysis of CCCC and CSCS Thin laminated rectangular composite plates. The polynomial series were used in generating plate deflection equations for the CCCC and CSCS conditions. Forces that can cause buckling on CCCC and CSCS were considered while applying aspect ratio to obtain critical buckling load. This work considered the aspect ratio in the range of one to two (1-2). The elastic modulus of different lamina that makes up the composite were considered. The laminas considered includes $0^0 0^0$ and $0^0 90^0$ for two laminas, $0^0 0^0 0^0$ and $0^0 90^0 0^0$ for three laminas, $0^0 0^0 0^0 0^0$ and $0^0 90^0 90^0 0^0$ for four laminas. The results showed that the CCCC has higher buckling coefficient than CSCS; also, the orientation affects the resistance to buckling.

Keywords: Total Potential Energy, Buckling, laminated composite, CCCC and CSCS.

1. INTRODUCTION

A composite material may consist of two or more lamina that have different properties and distinct end conditions between the components which together produces desirable properties that cannot be Obtained with any of the constituent alone. The concept of adding several laminas or components to produce a material with required properties that are not obtainable with an individual lamina component has been used by humans for thousands of years. (Reddy, 2004; Anya *et al.,* 2021; Ibearugbulam *et al*., 2020).

A lamina or ply may be defined as a flat thin layer or sheet which can be added to other materials to produce a composite structure. It can represent a fundamental building block in composite structure. It may also be seen as a macro unit of materials whose material properties are determined through appropriate laboratory test (Lin, 1973). According to Ibearugbulam., *et al*, (2021) Thin composite structure may fail in two ways which relates to each other. They include material failure and form failure. Material failure occur when the stresses in the structural component exceed the specified safe limit which the structure can carry resulting in the formation of several cracks which causes failure. For form failure to occur, it does not necessarily need the stresses that acts on the structure to exceed the safe value (limit state). It means that while the stresses are acting on the structure, though it has not exceeded the maximum permissible stress the structure may not be able to maintain its initial or original shape. Conclusively, in form failure the components of the structure do not fail by exceeding limit state rather by deforming into some other shape that is different from its initial shape due to intolerable external force or disturbance acting on the structure. (Ezeh *et al.*, 2013 and Ibrahim *et al.,* 2012).

There are some properties that form failure depends on; they include the material properties, geometrical properties and loading pattern on the structure. The material components include the elastic modulus, Poisson's ratio, fiber orientation, fiber volume fraction, and density. Loading type that can result to form failure include flexural loading, axial or compressive loading. Form failure that occurs as a result of introduction of compressive stresses are regarded as compressive loading. When the magnitude of such loading on a structure is such that the equilibrium moves from stable to neutral,

ISSN: 2992-3727 Published 30th November, 2025

the load is usually called the critical load. This process or phenomenon of change of equilibrium state is usually called the buckling of the structure (Iyengar,1998).

There are several ways of carrying out analysis of buckling of thin laminated composite plates but virtual all the methods are relied on assumption to obtain the displacement function that represent the deformed shape (Reddy, 2004; Singh and Chakarabarti, 2012; Fares, 1999; Han and Rus, 2016; Ferreira, et al 2004; Subramani and Sharmila, 2014). Despite the fact that the shape function was assumed some went along to make use of finite element method of analysis which is also an approximate method of structural analysis (Singh and Chakarabarti, 2012). The shape function assumed is either based on Navier method or levy method. The fact remains that any assumed shape displacement function will always produce results that will be different from the exact result except if the shape displacement function assumed is the exact displacement shape function. Ibearugbulem et al, 2020 provided exact displacement function for buckling of Simply-supported, simply-supported (SSSS) thin laminated composite plate but failed to provided ways of analyzing that of Clamped-Simply supported, clamped-Simply supported edge (CSCS) and clamped on the three edges and free on one edge (CCCF)

The present work intends to filled the gap identified by the use the Total potential Energy function to analyses thin CSCS and CFCF laminated composite plates.

2. METHODOLOGY

The method utilized in this analysis is the total potential energy function for buckling of thin laminated plate presented by Ibearugbulem et al 2020 and presented here as Equation (1) and Equation (2)

$$\Pi = \frac{1}{2}\iiint [\sigma][\varepsilon]\ dx\,.dy\,.dz - \frac{N_x}{2}\iint \left(\frac{dw}{dx}\right)^2\ dx\,dy \qquad (1)$$

$$\frac{N_x a^2}{D_1 t^3}k_N = E_1\left\{\left(\frac{D_{ii}}{D_1}k_x + \frac{2}{\alpha^2}\left(\frac{D_{ij}}{D_1} + 2\frac{D_{kk}}{D_1}\right)k_{xy} + \frac{D_{jj}}{D_1}.\frac{k_y}{\alpha^4} + 4\frac{D_{ik}}{D_{jj}}.\frac{k_{xxy}}{\alpha} + 4\frac{D_{jk}}{D_{jj}\alpha^3}k_{xyy}\right)\right.$$
$$-\frac{T_2}{D_1}\left(B_{ii}k_x + \frac{(B_{ij}+2B_{kk})}{\alpha^2}k_{xy} + \frac{3B_{ik}}{\alpha}k_{xxy} + \frac{B_{jk}}{\alpha^3}k_{xyy}\right)$$
$$\left.-\frac{T_3}{D_1}\left(\frac{(B_{ij}+2B_{kk})}{\alpha^2}k_{xy} + \frac{B_{jj}}{\alpha^4}k_y + \frac{B_{ik}}{\alpha}k_{xxy} + \frac{3B_{jk}}{\alpha^3}k_{xyy}\right)\right\} \qquad (2)$$

Equation (2) was presented by Anya *et al* 2021 as the general equation for analysis of thin rectangular laminated composite plate

Equation (2) when re-arranged gives will give Equation (3)

$$\frac{N_x a^2}{D_1 t^3} = \frac{N_x a^2}{\overline{D_1}} = E_1\left(\frac{k_{T1}+k_{T2}+k_{T3}}{k_N}\right) \qquad (3)$$

Where:

$$k_{T1} = \left(\frac{D_{ii}}{D_1}k_x + \frac{2}{\alpha^2}\left(\frac{D_{ij}}{D_1} + 2\frac{D_{kk}}{D_1}\right)k_{xy} + \frac{D_{jj}}{D_1}.\frac{k_y}{\alpha^4} + 4\frac{D_{ik}}{D_1}.\frac{k_{xxy}}{\alpha} + 4\frac{D_{jk}}{D_1\alpha^3}k_{xyy}\right) \qquad (4)$$

$$k_{T2} = -\frac{T_2}{D_1}\left(B_{ii}k_x + \frac{(B_{ij}+2B_{kk})}{\alpha^2}k_{xy} + \frac{4B_{ik}}{\alpha}k_{xxy} + \frac{B_{jk}}{\alpha^3}k_{xyy}\right) \qquad (5)$$

ISSN: 2992-3727 Published 30th November, 2025

$$k_{T3} = -\frac{T_3}{D_1}\left(\frac{(B_{ij} + 2B_{kk})}{\alpha^2}k_{xy} + \frac{B_{jj}}{\alpha^4}k_y + \frac{B_{ik}}{\alpha}k_{xxy} + \frac{4B_{jk}}{\alpha^3}k_{xyy}\right) \quad (6)$$

When the aspect ratio is considered as a/b and the parameters are in terms of long length "b" then the buckling equation is presented as

$$N_x b^2 = [N_x a^2] \times [b/a]^2 \quad (7)$$

The equation presented here as Equation (7) is a global form of that derived by Ibearugbulem *et al,* 2020, and it will be use with deflection equations generated using polynomial function for CCCC and CSCS during the numerical analysis to determine the critical buckling value for CCCC and CSCS different plate boundary conditions

3. Polynomial Equation for CCCC and CSCS

3.1 Derivation of the Deflection Equation for a CSCS Plate

Consider the CSCS plate shown in Figure 2, having the X(R) and Y(Q) as the two lengths of the plate

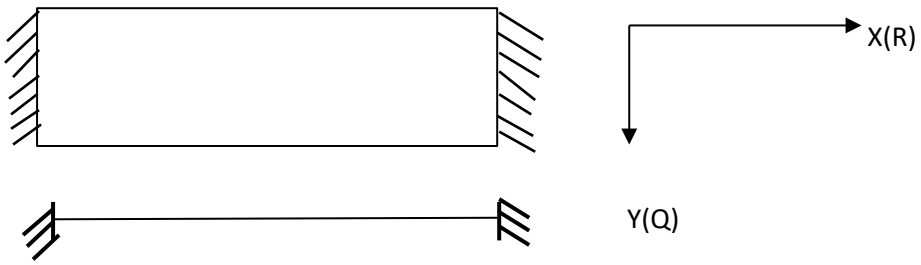


Figure 2: An CSCS Plate indicating X and Y direction. The deflection equation of CSCS is

$$w = a_4(R^2 - 2R^3 + R^4)b_4(Q - 2Q^3 + Q^4) = a_4 b_4(R^2 - 2R^3 + R^4)(Q - 2Q^3 + Q^4)$$

$$h = (R^2 - 2R^3 + R^4)(Q - 2Q^3 + Q^4) \quad (8)$$

$$K_x = \int_0^1 \int_0^1 \left(\frac{d^2h}{dR^2}\right)^2 dRdQ = \int_0^1 \int_0^1 (4 - 48R + 192R^2 - 288R^3 + 144R^4)(Q^2 - 4Q^4 + 2Q^5 + 4Q^6 - 4Q^7 + Q^8)\, dRdQ \quad (9)$$

$$K_{xy} = \int_0^1 \int_0^1 \left(\frac{d^2h}{dRdQ}\right)^2 dRdQ = \int_0^1 \int_0^1 (4R^2 - 24R^3 + 52R^4 - 48R^5 + 16R^6)(1 + 12Q^2 + 8Q^3 + 36Q^4 - 48Q^5 + 16Q^6)\, dRdQ \quad (10)$$

$$K_y = \int_0^1 \int_0^1 (R^4 - 4R^5 + 6R^6 - 4R^7 + R^8)(144Q^4 - 288Q^3 + 144Q^2)\, dRdQ$$

ISSN: 2992-3727 Published 30th November, 2025

$$K_{xxy} = \int_0^1 \int_0^1 \frac{d^3h}{dR^3} \cdot \frac{dh}{dQ} dRdQ$$
$$= \int_0^1 \int_0^1 (-12R + 24R^2 + 24R^3 - 60R^4 + 24R^5)(Q - 8Q^3 + 4Q^4 + 12Q^5 - 14Q^6 + 4Q^7)\, dRdQ \quad (11)$$

$$k_{xyy} = \int_0^1 \int_0^1 \frac{dh}{dR} \frac{d^3h}{dQ^3} dRdQ$$
$$= \int_0^1 \int_0^1 (2R^3 - 10R^4 + 18R^5 - 14R^6 + 4R^7)(-12Q + 24Q^2 + 24Q^3 - 36Q^4 + 24Q^5)\, dRdQ \quad (12)$$

$$K_N = \int_0^1 \int_0^1 \left(\frac{dh}{dR}\right)^2 dRdQ = \int_0^1 \int_0^1 \left((2R - 6R^2 + 4R^3)^2((Q - 2Q^3 + Q^4)^2)\right) dRdQ \quad (13)$$

3.2 Derivation of the Deflection Equation for a CCCC Plate

Consider the CCCC plate shown in Figure 3, having the X(R) and Y(Q) as the two lengths of the plate

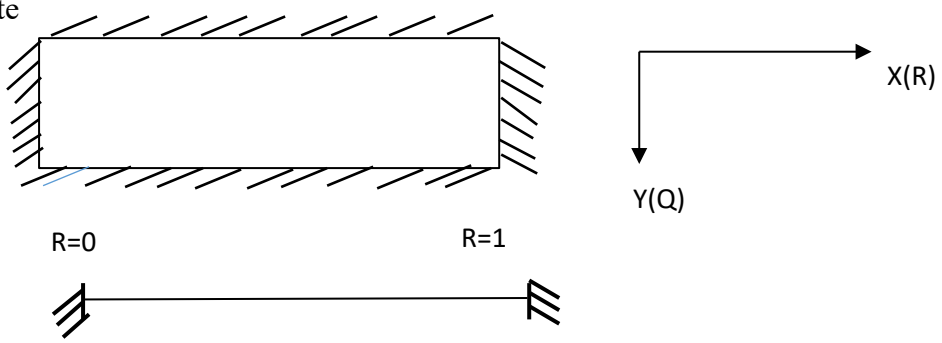


Figure 3: An CCCF Plate indicating X and Y direction

The deflection equation of CCCF is
Equation for deflection for CCCF is as shown in Equation (14)

$$w = w_x w_y = a_4(R^2 - 2R^3 + R^4)b_4(Q^2 - 2Q^3 + Q^4)$$
$$W = C_1 h \; where \; C_1 = a_4 b_4$$
$$h = (R^2 - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4) \quad (14)$$

$$K_x = \int_0^1 \int_0^1 \left(\frac{d^2h}{dR^2}\right)^2 dRdQ$$
$$= \int_0^1 \int_0^1 \left((4 - 48R + 192R^2 - 288R^3 + 144R^4)(Q^4 - 4Q^5 + 6Q^6 - 4Q^7 + Q^8)\right) dRdQ \quad (15)$$

$$K_{xy} = \int_0^1 \int_0^1 \left(\frac{d^2h}{dRdQ}\right)^2 dRdQ$$
$$= \int_0^1 \int_0^1 (4R^2 - 24R^3 + 52R^4 - 48R^5 + 16R^6)(4Q^2 - 24Q^3 + 52Q^4 - 48Q^5 + 16Q^6)\, dRdQ \quad (16)$$

$$K_y = \int_0^1 \int_0^1 \left(\frac{d^2h}{dQ^2}\right)^2 dRdQ = \int_0^1 \int_0^1 \left((R^4 - 4R^5 + 6R^6 - 4R^7 + R^8)(4 - 48Q + 192Q^2 - 288Q^3 + 144Q^4)\right) dRdQ \quad (17)$$

$$K_{xxy} = \int_0^1 \int_0^1 \frac{d^3h}{dR^3}.\frac{dh}{dQ} dRdQ = \int_0^1 \int_0^1 (-12 + 24R)(Q^2 - 2Q^3 + Q^4)(R^2 - 2R^3 + R^4)(2Q - 6Q^2 + 4Q^3) dRdQ \quad (18)$$

$$k_{xyy} = \int_0^1 \int_0^1 \frac{dh}{dR}\frac{d^3h}{dQ^3} dRdQ = \int_0^1 \int_0^1 (R^2 - 2R^3 + R^4)(-12 + 24Q)(2R - 6R^2 + 4R^3)(Q^2 - 2Q^3 + Q^4)\, dRdQ \quad (19)$$

$$K_N = \int_0^1 \int_0^1 [\![\left((4R^2 - 24R^3 + 52R^4 - 48R^5 + 16R^6)\right)\left((Q^4 - 4Q^5 + 6Q^6 - 4Q^7 + Q^8)\right)]\!] \quad (20)$$

4. Numerical Example

Consider a rectangular thin laminated composite plate with orientation angles $0^0 0^0$ and $0^0 90^0$, for two laminates, $0^0 0^0 0^0$ and $0^0 90^0 0^0$, for three laminates and $0^0 0^0 0^0 0^0$ and $0^0 90^0 90^0 0^0$ for four laminates with the following boundary conditions CSCS (simply supported in opposite edges and clamped in the remaining edges) and CCCC (Clamped at all edges) having aspect ratio ranging from 1 to 2. The composite plates have the following material properties $E_1/E_2 = 25$; $G_{12}/E_2 = 0.5$; $V_{12} = 0.25$. For the above composite plates, it is required to obtain the critical buckling coefficient for the plate.

Let the reference elastic modulus E_1, be taken as E_2. Hence

$$\frac{N_x a^2}{D_1 t^3} = \frac{N_x a^2}{\overline{D_1}} = E_1\left(\frac{k_{T1} + k_{T2} + k_{T3}}{k_N}\right) \quad (3)$$

The coefficients of stiffness obtained using the polynomial deflection function above are as shown on Table 1

Table 1: Coefficients of stiffness (k-values) for CSCS and CCCC

End Condition	K_x	K_{xy}	K_y	K_{xxy}	K_{xyy}	K_N
CSCS	0.039365	0.009233	0.007619	0	0	0.000937
CCCC	0.00127	0.000361	0.00127	0	0	0.00003

For CCCC plate

$$\frac{N_x a^2}{D_1 t^3} = \frac{N_x a^2}{\overline{D_1}} = E_1\left(\frac{k_{T1} + k_{T2} + k_{T3}}{k_N}\right)$$

ISSN: 2992-3727 Published 30th November, 2025

$$k_{T1} = \left(\frac{D_{11}}{D_0}(0.00127) + \frac{2}{\alpha^2}\left(\frac{D_{12}}{D_0} + 2\frac{D_{33}}{D_0}\right)(0.000361) + \frac{D_{22}}{D_0}.\frac{(0.00127)}{\alpha^4} + 4\frac{D_{13}}{D_0}.\frac{(0)}{\alpha} + 4\frac{D_{23}}{D_0\alpha^3}(0)\right)$$

$$k_{T1} = \left(\frac{D_{11}}{D_0}(0.00127) + \frac{2}{\alpha^2}\left(\frac{D_{12}}{D_0} + 2\frac{D_{33}}{D_0}\right)(0.000361) + \frac{D_{22}}{D_0}.\frac{(0.00127)}{\alpha^4}\right) \quad (21)$$

$$k_{T2} = -\frac{T_2}{D_0}\left(B_{11}k_x + \frac{(B_{12}+2B_{33})}{\alpha^2}k_{xy} + \frac{4B_{13}}{\alpha}k_{xxy} + \frac{B_{23}}{\alpha^3}k_{xyy}\right)$$

$$k_{T2} = -\frac{T_2}{D_0}\left(B_{11}(0.00127) + \frac{(B_{12}+2B_{33})}{\alpha^2}(0.000361) + \frac{4B_{13}}{\alpha}(0) + \frac{B_{23}}{\alpha^3}(0)\right)$$

$$k_{T2} = -\frac{T_2}{D_0}\left(B_{11}(0.00127) + \frac{(B_{12}+2B_{33})}{\alpha^2}(0.000361)\right) \quad (22)$$

$$k_{T3} = -\frac{T_3}{D_0}\left(\frac{(B_{12}+2B_{33})}{\alpha^2}k_{xy} + \frac{B_{22}}{\alpha^4}k_y + \frac{B_{13}}{\alpha}k_{xxy} + \frac{4B_{23}}{\alpha^3}k_{xyy}\right)$$

$$k_{T3} = -\frac{T_3}{D_0}\left(\frac{(B_{12}+2B_{33})}{\alpha^2}(0.000361) + \frac{B_{22}}{\alpha^4}(0.00127) + \frac{B_{13}}{\alpha}(0) + \frac{4B_{23}}{\alpha^3}(0)\right)$$

$$k_{T3} = -\frac{T_3}{D_0}\left(\frac{(B_{12}+2B_{33})}{\alpha^2}(0.000361) + \frac{B_{22}}{\alpha^4}(0.00127)\right) \quad (23)$$

For the orientation of $0^0 0^0$

Table 2: Values of z for two laminas at any orientation

For Two laminas

	Z_0	Z_1	Z_2
	0.5	0	-0.5
Lamina 1	$Z_{n-1} - Z_n$	$Z_{n-1}^2 - Z_n^2$	$Z_{n-1}^3 - Z_n^3$
	0.5	0.25	0.125
Lamina 2	$Z_{n-1} - Z_n$	$Z_{n-1}^2 - Z_n^2$	$Z_{n-1}^3 - Z_n^3$
	0.5	-0.25	0.125

The elastic modulus of each lamina becomes

ISSN: 2992-3727 Published 30th November, 2025

$$[E_{12}]1 = \begin{bmatrix} 25.0626 & 0.250626 & 0 \\ 0.250626 & 1.00 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}; \quad [E_{12}]2 = \begin{bmatrix} 25.0626 & 0.250626 & 0 \\ 0.250626 & 1.00 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}$$

Solving to obtain values **of** $A_{11}, A_{12}, A_{13}, A_{21}, A_{22}, B_{11}, B_{12}, B_{13}, B_{21}, B_{22},$

$D_{11}, D_{12}, D_{13}, D_{21}, D_{22}$ Having in mind that

$$A_{11} = (z_{n-1} - z_n)(m^4 e_{11} + 2m^2 n^2 (e_{12} + 2e_{33}) + n^4 e_{22}) = (z_{n-1} - z_n) a_{11} \quad (24)$$

$$A_{12} = (z_{n-1} - z_n)(e_{12}(n^4 + m^4) + m^2 n^2 (e_{11} + e_{22} - 4e_{33})) = (z_{n-1} - z_n) a_{12} \quad (25)$$

$$A_{13} = (z_{n-1} - z_n)(m^3 n(e_{11} - e_{12} - 2e_{33}) + mn^3 (e_{12} - e_{22} + 2e_{33}))$$

$$= (z_{n-1} - z_n) a_{13} \quad (26)$$

$$A_{22} = (z_{n-1} - z_n)(n^4 e_{11} + 2m^2 n^2 (e_{12} + 2e_{33}) + m^4 e_{22}) = (z_{n-1} - z_n) a_{22} \quad (27)$$

$$A_{23} = (z_{n-1} - z_n)\big(mn^3 e_{11} - m^3 n e_{22} + (m^3 n - mn^3)(e_{12} + 2e_{33})\big)$$

$$= (z_{n-1} - z_n) a_{23} \quad 28)$$

$$A_{33} = (z_{n-1} - z_n)(m^2 n^2 (e_{11} - 2e_{12} + e_{22} - 2e_{33}) + e_{33}(m^4 + n^4))$$

$$= (z_{n-1} - z_n) a_{33} \quad (29)$$

$A_{21} = A_{12},\ A_{31} = A_{13}\ and\ A_{32} = A_{23}$ where $m = Cos\ 0 = 1, n = Sin\ 0 = 0$

This implies that lamina one and two will have axial stiffness as expressed in Equation (30)

$$\begin{aligned}
A_{11} &= (0.5)\Big((1)(25.0626) + 2(0)\big(0.250626 + 2(0.5)\big) + (0)(1)\Big) = 12.531 \\
A_{12} &= (0.5)((0.250626)(0+1) + 0(25.0626 + 1 - 4(0.5))) = 0.1253 \\
A_{13} &= (0.5)0\big(25.0626 - 0.250626 - 2(0.5)\big) + 0\big(0.250626 - 1.0 + 2(0.5)\big)) \\
A_{13} &= 0.000 \\
A_{22} &= (0.5)\big(0 * 25.0626 + 2(0)\big(0.250626 + 2(0.5)\big) + 1.0 * 1.0\big) = 0.5013 \\
A_{23} &= (0.5)\big((0)25.0626 - 0 * 1.0 + (0-0)(0.250626 + 2 * 0.5)\big) = 0.000 \\
A_{33} &= (0.5)\big(0 * (25.0626 - 2 * 0.250626 + 1.0 - 2 * 0.5) + 0.5(1.0 + 0)\big) \\
A_{33} &= 0.2500 \\
A_{21} &= A_{12},\ A_{31} = A_{13}\ and\ A_{32} = A_{23}
\end{aligned} \quad (31)$$

The values for coupling stiffness coefficient B were as obtained in Equation (32a)-Equation (3.541f)

$$B_{11} = \left(\frac{{z_{n-1}}^2 - {Z_n}^2}{2}\right)(m^4 e_{11} + 2m^2 n^2(e_{12} + 2e_{33}) + n^4 e_{22}) \quad (32a)$$

$$B_{12} = \left(\frac{{z_{n-1}}^2 - {Z_n}^2}{2}\right)(e_{12}(n^4 + m^4) + m^2 n^2(e_{11} + e_{22} - 4e_{33}) \quad (32b)$$

$$B_{13} = \left(\frac{{z_{n-1}}^2 - {Z_n}^2}{2}\right)(m^3 n(e_{11} - e_{12} - 2e_{33}) + mn^3(e_{12} - e_{22} + 2e_{33}) \quad (32c)$$

$$B_{22} = \left(\frac{{z_{n-1}}^2 - {Z_n}^2}{2}\right)(n^4 e_{11} + 2m^2 n^2(e_{12} + 2e_{33}) + m^4 e_{22}) \quad (32d)$$

$$B_{23} = \left(\frac{{z_{n-1}}^2 - {Z_n}^2}{2}\right)(mn^3 e_{11} - m^3 n e_{22} + (m^3 n - mn^3)(e_{12} + 2e_{33}) \quad (32e)$$

$$B_{33} = \left(\frac{{z_{n-1}}^2 - {Z_n}^2}{2}\right)(m^2 n^2(e_{11} - 2e_{12} + e_{22} - 2e_{33}) + e_{33}(m^4 + n^4) \quad (32f)$$

$B_{21} = B_{12}, \quad B_{31} = B_{13} \ and \ B_{32} = B_{23}$

This implies that for lamina one, coupling stiffness coefficient B values will be as expressed in

Equations (33)

$$B_{11} = (0.125)((1)(25.0626) + 2(0)\big(0.250626 + 2(0)\big) + 0 * 1.003) = 3.1328$$

$$B_{12} = (0.125)\left((0.250626)(0+1) + 0\big(25.0626 + 1 - 4(0.5)\big)\right) = 0.0313$$

$$B_{13} = (0.125)0\big(25.0626 - 0.250626 - 2(0.5)\big) + 0\big(0.250626 - 1.0 + 2(0.5)\big)$$

$$B_{13} = 0.0000$$

$$B_{22} = (0.125)\big(0 * 25.0626 + 2(0)\big(0.250626 + 2(0.5)\big) + 1.0 * 1.0\big) = 0.1253$$

$$B_{23} = (0.125)\big((0)25.0626 - 0 * 1.0 + (0-0)(0.250626 + 2 * 0.5)\big) = 0.0000$$

$$B_{33} = (0.125)\big(0 * (25.0626 - 2 * 0.250626 + 1.0 - 2 * 0.5) + 0.5(1.0 + 0)\big)$$

$$B_{33} = 0.0625 \ B_{21} = B_{12}, \quad B_{31} = B_{13} \ and \ B_{32} = B_{23}$$

(33a)

This implies that for lamina two, coupling stiffness coefficient B values will be as expressed in Equations (33b)

$$B_{11} = (-0.125)\left((1)(25.0626) + 2(0)\big(0.250626 + 2(0)\big) + 0 * 1.003\right)$$

$B_{11} = -3.1328$

$B_{12} = (-0.125)\left((0.250626)(0+1) + 0\left(25.0626 + 1 - 4(0.5)\right)\right) = -0.0313$

$B_{13} = (-0.125)0\left(25.0626 - 0.250626 - 2(0.5)\right) + 0\left(0.250626 - 1.0 + 2(0.5)\right)$

$B_{13} = 0.0000$

$B_{22} = (-0.125)\left(0 * 25.0626 + 2(0)\left(0.250626 + 2(0.5)\right) + 1.0 * 1.0\right) = -0.1253$

$B_{23} = (-0.125)\left((0)25.0626 - 0 * 1.0 + (0-0)(0.250626 + 2 * 0.5)\right) = 0.0000$

$B_{33} = (-0.125)\left(0 * (25.0626 - 2 * 0.250626 + 1.0 - 2 * 0.5) + 0.5(1.0 + 0)\right)$

$B_{33} = -0.0625$

$B_{21} = B_{12}, \quad B_{31} = B_{13} \ and \ B_{32} = B_{23}$

The values for bending stiffness coefficient, D were as expressed in Equations (34a) to Equation (3.543f)

$$D_{11} = \left(\frac{{z_{n-1}}^3 - {Z_n}^3}{3}\right)(m^4 e_{11} + 2m^2 n^2 (e_{12} + 2e_{33}) + n^4 e_{22}) \qquad (34)$$

$$D_{12} = \left(\frac{{z_{n-1}}^3 - {Z_n}^3}{3}\right)(e_{12}(n^4 + m^4) + m^2 n^2 (e_{11} + e_{22} - 4e_{33}) \qquad (34b)$$

$$D_{13} = \left(\frac{{z_{n-1}}^3 - {Z_n}^3}{3}\right)(m^3 n(e_{11} - e_{12} - 2e_{33}) + mn^3 (e_{12} - e_{22} + 2e_{33}) \quad (34c)$$

$$D_{22} = \left(\frac{{z_{n-1}}^3 - {Z_n}^3}{3}\right)(n^4 e_{11} + 2m^2 n^2 (e_{12} + 2e_{33}) + m^4 e_{22}) \qquad (34d)$$

$$D_{23} = \left(\frac{{z_{n-1}}^3 - {Z_n}^3}{3}\right)(mn^3 e_{11} - m^3 n e_{22} + (m^3 n - mn^3)(e_{12} + 2e_{33}) \quad (34e)$$

$$D_{33} = \left(\frac{{z_{n-1}}^3 - {Z_n}^3}{3}\right)(m^2 n^2 (e_{11} - 2e_{12} + e_{22} - 2e_{33}) + e_{33}(m^4 + n^4) \quad (34f)$$

$D_{21} = D_{12}, \quad D_{31} = D_{13} \ and \ D_{32} = D_{23}$

This implies that the values for bending stiffness coefficient D for lamina one and two are as expressed in Equation (35)

$D_{11} = (0.042)\left((1)(25.0626) + 2(0)\left(0.250626 + 2(0)\right) + 0 * 1.003\right)$

$D_{11} = 1.0443$ (35)

ISSN: 2992-3727 Published 30th November, 2025

$$D_{12} = (0.042)\Big((0.250626)(0+1) + 0\big(25.0626 + 1 - 4(0.5)\big)\Big) = 0.0104$$

$$D_{13} = (0.042)0\big(25.0626 - 0.250626 - 2(0.5)\big) + 0\big(0.250626 - 1.0 + 2(0.5)\big))$$

$$D_{13} = 0.00$$

$$D_{22} = (0.042)\big(0 * 25.0626 + 2(0)\big(0.250626 + 2(0.5)\big) + 1.0 * 1.0\big) = 0.0418$$

$$D_{23} = (0.042)\big((0)25.0626 - 0 * 1.0 + (0-0)(0.250626 + 2 * 0.5)\big)) = 0.00$$

$$D_{33} = (0.042)\big(0 * (25.0626 - 2 * 0.250626 + 1.0 - 2 * 0.5) + 0.5(1.0 + 0)\big)$$

$$D_{33} = 0.0208$$

$$D_{21} = D_{12}, \qquad D_{31} = D_{13} \ \ and \ \ D_{32} = D_{23}$$

ISSN: 2992-3727 Published 30th November, 2025

Also Solving Equations (3.39), (3.38a), and (3.38b) gives Equations (36), (37) and (38)

$$d_{11} = 25.0627 * 0.2362 + 0.5000 * \frac{0.2359}{\alpha^2} + 2 * 0.0000 * \frac{0}{\alpha}$$

$$d_{12} = 0.2506 * \frac{0.2359}{\alpha^2} + 0.5000 * \frac{0.2359}{\alpha^2} + 0.0000 * \frac{0}{\alpha} + A_{23}\frac{0}{\alpha^3}$$

$$d_{22} = 1.0025 * \frac{0.2362}{\alpha^4} + 0.5000 * \frac{0.2359}{\alpha^2} + 2 * 0.00 * \frac{0}{\alpha^3}$$

$$d_{13} = 0 * 0.2362 + (0 + 2 * 0)\frac{0.2359}{\alpha^2} + 4 * 0 * \frac{0}{\alpha} + 0 * \frac{0}{\alpha^3} = 0$$

$$d_{23} = 0 * \frac{0.2362}{\alpha^4} + (0 + 2 * 0)\frac{0.2359}{\alpha^2} + 0 * \frac{0}{\alpha} + 4 * 0\frac{0}{\alpha^3} = 0 \qquad (36)$$

$$while\ T_2 = \frac{(d_{12}.d_{23} - d_{13}.d_{22})}{\left(d_{12}{}^2 - d_{11}d_{22}\right)} = 0, \qquad (37)$$

$because\ \ d_{13}\ and\ \ d_{23}\ are\ equal\ to\ zero$

$$T_3 = \frac{(d_{12}.d_{13} - d_{11}d_{23})}{\left(d_{12}{}^2 - d_{11}d_{22}\right)} = 0, \qquad (38)$$

$because\ \ d_{13}\ and\ \ d_{23}\ are\ equal\ to\ zero$

The combination of the material properties gives Table 3

ISSN: 2992-3727 Published 30th November, 2025

Table 3: computation of Elastic modulus parameters for two laminas with orientation 0^0, 0^0

	a_{11}	a_{12}	a_{13}	a_{22}	a_{23}	a_{33}	$a_{xy} = (a_{12} + 2b_{33})$	$(z_{n-1} - z_n)$	$\left(\frac{z_{n-1}^2 - Z_n^2}{2}\right)$	$\left(\frac{z_{n-1}^3 - Z_n^3}{3}\right)$
m1	25.06	0.250	0	1.00	0	0.5	1.25	0.5	0.125	0.042
m2	25.06	0.250	0	1.00	0	0.5	1.25	0.5	-0.125	0.042

	$\mathbf{A_{11}}$	$\mathbf{A_{12}}$	$\mathbf{A_{13}}$	$\mathbf{A_{22}}$	$\mathbf{A_{23}}$	$\mathbf{A_{33}}$	$\mathbf{A_{xy} = (A_{12} + 2A_{33})}$
m1	12.5313	0.1253	0.0000	0.5013	0.0000	0.2500	0.6253
m2	12.5313	0.1253	0.0000	0.5013	0.0000	0.2500	0.6253
sum	25.0627	0.2506	0.0000	1.0025	0.0000	0.5000	1.2506

	$\mathbf{B_{11}}$	$\mathbf{B_{12}}$	$\mathbf{B_{13}}$	$\mathbf{B_{22}}$	$\boldsymbol{B_{23}}$	$\mathbf{B_{33}}$	$\mathbf{B_{xy} = (B_{12} + 2B_{33})}$
m1	3.1328	0.0313	0.0000	0.1253	0.0000	0.0625	0.1563
m2	-3.1328	-0.0313	0.0000	-0.1253	0.0000	-0.0625	-0.1563
sum	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

	$\mathbf{D_{11}}$	$\mathbf{D_{12}}$	$\mathbf{D_{13}}$	$\mathbf{D_{22}}$	$\mathbf{D_{23}}$	$\mathbf{D_{33}}$	$\mathbf{D_{xy} = (D_{12} + 2D_{33})}$
m1	1.0443	0.0104	0.0000	0.0418	0.0000	0.0208	0.0521
m2	1.0443	0.0104	0.0000	0.0418	0.0000	0.0208	0.0521
Sum	2.0886	0.0209	0.0000	0.0835	0.0000	0.0417	0.1042

For $\alpha = 1$

$\boldsymbol{\phi_{11}}$	$\boldsymbol{\phi_{12}}$	$\boldsymbol{\phi_{13}}$	$\boldsymbol{\phi_{22}}$	$\boldsymbol{\phi_{23}}$	$\boldsymbol{\phi_{33}}$
25.0000	0.2500	0.0000	1.0000	0.0000	0.4988

$\boldsymbol{d_{11}}$	$\boldsymbol{d_{12}}$	$\boldsymbol{d_{22}}$	$\boldsymbol{d_{13}}$	$\boldsymbol{d_{23}}$	**T2**	**T3**
6.0377	0.1771	0.3547	0.0000	0.0000	0.0000	0.0000

Where M_1 and M_2 are the two laminas of the composite plate

ISSN: 2992-3727 Published 30th November, 2025

From Table 3.3

$$k_{T1} = \left(\frac{(2.0886)}{0.0835}(0.00127) + \frac{2}{\alpha^2}\left(\frac{(0.0209)}{0.0835} + 2\frac{(0.0417)}{0.0835}\right)(0.000361) + \frac{(0.0835)}{0.0835}.\frac{(0.00127)}{\alpha^4}\right) \quad (39a)$$

$$k_{T2} = -\frac{(0)}{0.0835}\left(B_{11}(0.00127) + \frac{(B_{12}+2B_{33})}{\alpha^2}(0.000361\right) \quad (39b)$$

$$k_{T3} = -\frac{(0)}{0.0835}\left(\frac{(B_{12}+2B_{33})}{\alpha^2}(0.000361 + \frac{B_{22}}{\alpha^4}(0.00127)\right) \quad (39c)$$

Therefore, Equation (3) becomes Equation (40)

$$\frac{N_x a^2}{\overline{D_{22}}} = 1\left(\frac{\left(\frac{(2.0886)}{0.0835}(0.00127) + \frac{2}{\alpha^2}\left(\frac{(0.0209)}{0.0835} + 2\frac{(0.0417)}{0.0835}\right)(0.000361) + \frac{(0.0835)}{0.0835}.\frac{(0.00127)}{\alpha^4}\right)}{0.00003}\right) \quad (40)$$

Simplifying Equation (40) yields Equation (40b)

$$\frac{N_x a^2}{D_{22}} = \frac{1}{0.0835}\left(\frac{\left((0.002653) + \frac{1}{\alpha^2}(0.04186) + \frac{(0.00011)}{\alpha^4}\right)}{0.00003}\right) \quad (40b)$$

For aspect ration $\alpha = 1$, Equation (3.718b) gives (3.718c)

$$\frac{N_x a^2}{\overline{D_{22}}} = \frac{1}{0.0835}\left(\frac{\left((0.002653) + \frac{1}{1^2}(0.04186) + \frac{(0.00011)}{1^4}\right)}{0.00003}\right) \quad (40c)$$

5.0 Results, Discussion and Conclusion

The results are as presented on Table 4 and Table 5. From Table 4, it can be seen that the Buckling coefficient for $0^0 0^0$, $0^0 0^0 0^0$ and $0^0 0^0 0^0 0^0$ are approximately the same but decreases as the aspect ratio increases. The values of buckling coefficient for $0^0 90^0$ is very smaller than those of $0^0 0^0$, $0^0 0^0 0^0$ and $0^0 0^0 0^0 0^0$. The coefficient obtained from $0^0 90^0 90^0 0^0$ ranges from 24.858 to 23.619 for aspect ratio of 1 to 2 respectively. The values of $0^0 90^0 0^0$ decreases with increase in aspect ratio. The change in orientation makes the plate to be more resistance to buckling. The more the number of laminas arranged in different angle the more resistance to buckling.

Table 4: Orientation Angle and Buckling Coefficient for CSCS Plates

Orientations	$0^0 0^0$	$0^0 90^0$	$0^0 0^0 0^0$	$0^0 90^0 0^0$	$0^0 0^0 0^0 0^0$	$0^0 90^0 90^0 0^0$
Aspect ratio	$\frac{N_x a^2}{\pi^2 \overline{D_2}}$	$\frac{N_x a^2}{\pi^2 \overline{D_2}}$	$\frac{N_x a^2}{\pi^2 \overline{D_2}}$	$\frac{N_x a^2}{\pi^2 \overline{D_2}}$	$\frac{N_x a^2}{\pi^2 \overline{D_2}}$	$\frac{N_x a^2}{\pi^2 \overline{D_2}}$
1	109.732	2.001	109.732	56.478	109.732	24.858
1.1	109.038	1.877	109.038	55.988	109.038	24.489
1.2	108.544	1.796	108.544	55.648	108.544	24.242
1.3	108.179	1.739	108.179	55.404	108.179	24.069
1.4	107.902	1.699	107.902	55.223	107.902	23.944
1.5	107.687	1.669	107.687	55.084	107.687	23.851
1.6	107.516	1.647	107.516	54.976	107.516	23.781
1.7	107.378	1.630	107.378	54.890	107.378	23.726
1.8	107.264	1.616	107.264	54.821	107.264	23.682
1.9	107.170	1.605	107.170	54.764	107.170	23.647
2.0	107.091	1.597	107.091	54.716	107.091	23.619

Table 5: Orientation Angle and Buckling Coefficient for CCCC Plate

Orientations	$0^0 0^0$	$0^0 90^0$	$0^0 0^0 0^0$	$0^0 90^0 0^0$	$0^0 0^0 0^0 0^0$	$0^0 90^0 90^0 0^0$
Aspect ratio	$\frac{N_x a^2}{\pi^2 \overline{D_2}}$	$\frac{N_x a^2}{\pi^2 \overline{D_2}}$	$\frac{N_x a^2}{\pi^2 \overline{D_2}}$	$\frac{N_x a^2}{\pi^2 \overline{D_2}}$	$\frac{N_x a^2}{\pi^2 \overline{D_2}}$	$\frac{N_x a^2}{\pi^2 \overline{D_2}}$
1	114.554	3.300	114.554	60.646	114.525	28.639
1.1	112.668	2.774	112.668	59.008	112.668	27.147
1.2	111.407	2.436	111.407	57.935	111.407	26.186
1.3	110.528	2.211	110.528	57.203	110.528	25.541
1.4	109.896	2.055	109.896	56.687	109.896	25.094
1.5	109.427	1.944	109.427	56.312	109.427	24.775
1.6	109.071	1.864	109.071	56.033	109.071	24.542
1.7	108.795	1.758	108.795	55.820	108.795	24.367
1.8	108.576	1.759	108.576	55.655	108.576	24.234
1.9	108.401	1.723	108.401	55.525	108.401	24.130
2.0	108.258	1.696	108.258	55.421	108.258	24.049

From Table 4 and Table 5, it is obvious that the Buckling coefficient for $0^0 0^0$, $0^0 0^0 0^0$ and $0^0 0^0 0^0 0^0$ which are orthotropic materials have approximately the same value which decreases as the aspect ratio moves from 1 to 2. The coefficient of buckling for $0^0 90^0$ is very smaller than those of $0^0 0^0$, $0^0 0^0 0^0$ and $0^0 0^0 0^0 0^0$. The coefficient of buckling obtained from $0^0 90^0 90^0 0^0$ ranges from 28.63 to 24.049 for aspect ratio of 1 to 2 respectively. The values of $0^0 90^0 0^0$ decreases with increase in the aspect ratio from 1 to 2. The change in orientation makes the plate to be more resistance to buckling. The more the number of laminas arranged in different angle the more resistance to buckling.

6. CONFLICT OF INTEREST

The work has no conflict of interest and it was not sponsored by any organization.

ISSN: 2992-3727 Published 30th November, 2025

References

Anya, U.C. M., Okoroafor, S.U., Anyadiegwu P.C & Jorge, Ibearugbulem, O.M. (2021). Effect of Orientation Angles and Boundary Conditions on the Pure Bending Coefficient of Thin Laminated Composite Plate using Euler Bernoulli Equilibrium Equation. Nigerian Journal of Technology (NIJOTECH), Vol. 40, Pp. 999 –1009.

Ferreira, A. J. M., Roque, C. M. C., & Jorge, R. M. N. (2005). Analysis of composite plates by trigonometric shear deformation theory and multiquadrics. Computers and Structures (Elsevier Science Ltd), Vol. 83, Pp. 2225–2237.

Han, S., Lee, S., & Rus, G. (2006). Post buckling analysis of laminated composite plates subjected to the combination of in-plane shear, compression and lateral loading. International Journal of Solids and Structures (Elsevier Science Ltds) Vol 43, Pp. 5713–5735.

Ibearugbulem, O.M., Anya, U.C, Okoroafor, S.U., & Ezeh, J.C. (2020). Buckling Analysis of Thin Laminated Composite Plates with Exact Displacement Functions. Elixir Civil Engg. Vol. 142. No. 54362-54368.

Ibearugbulem, O. M., Osadebe, N. N., Ezeh, J. C., & Onwuka, D.O. (2011). Buckling analysis of axially compressed SSSS thin rectangular plate using Taylor-Mclaurin shape function. International Journal of Civil and Structural Engineering Volume 2, No 2.

Ibrahim, S.M., Carrera, E., Petrolo, M. & Zappino. E (2012). Buckling of composite thin-walled beams by refined theory. Composite Structures Vol 94, Pp 563-570.

Iyengar (1988). Structural Stability of Columns and plates. Ellis Horwood Limited Publishers Chichester, Pp. 225-252.

Kubiak, T (2013). Static and Dynamic Buckling of Thin-Walled Plate Structures. Springer International Publishing Switzerland. DOI: 10.1007/978-3-319-00654-3_2.

Lin, C. (1973). Free Transverse Vibration of Rectangular laminated plates. A doctorate degree project submitted to the Faculty of Division of Graduate Studies and Research Georgia Institute of Technology.

Osman, M.E.S, Osama, M.Y., & Tagelsir, H. (2019). Effect of Boundary Conditions on Buckling Load for Laminated Composite Plates. Glob J Sci. Vol 2, Iss.1. Issn 2641-2039. DOI: 10.33552/GJES.2019.02.000527.

Reddy, J. N. (2004). Mechanics of laminated composite plates and shells theories and analysis (2nd Ed.). London: CRC Press LLC ISBN: 0849315921, Pp 297-375.

Subramani, T., & Sharmila, S. (2014). Prediction of Deflection and Stresses of Laminated Composite Plate with Artificial Neural Network Aid. International Journal of Modern Engineering Research (IJMER). Vol. 4, Iss. 6, Pp. 51-58.